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Grand Unified Theories From Superstrings

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Abstract

I review how traditional grand unified theories, which require adjoint (or higher representation) Higgs fields for breaking to the standard model, can be contained within string theory. The status (as of January 1996) of the search for stringy free fermionic three generation $SO(10)$ SUSY-GUT models is discussed. Progress in free fermionic classification of both $SO(10)_2$ charged and uncharged embeddings and in $N = 1$ spacetime solutions is presented.

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SUSY–GUTs and Strings

Elementary particle physics has achieved phenomenal success in recent decades, resulting in the Standard Model (SM), $SU(3)_C \times SU(2)_L \times U(1)_Y$, and verification to high precision of many SM predictions. However, many aspects of the SM point to a more fundamental, underlying theory:

- the SM is very complicated, requiring measurement of some 19 free parameters,
- the SM has a complicated gauge structure,
- there is a naturalness problem regarding the scale of electroweak breaking,
- fine-tuning is required for the strong CP problem, and
- the expected cosmological constant resulting from electroweak breaking is many, many orders of magnitude higher than the experimental limit.

Since the early 1980's, these issues have motivated investigation of Grand Unified Theories (GUTs) that would unite SM physics through a single force at higher temperatures. Superstring research[1] has attempted to proceed one step further and even merge SM physics with gravity into a “Theory of Everything.”

Perhaps the most striking evidence for a symmetry beyond the SM is the predicted coupling unification not for the SM, but for the minimal supersymmetric standard model (MSSM) containing two Higgs doublets.[2] Renormalization group equations applied to the SM couplings measured around the M_{Z^0} scale predict MSSM unification at $M_{\text{unif}} \approx 2.5 \times 10^{16}$ GeV. However, this naively poses a problem for string theory, since the string unification scale has been computed, at tree level, to be one order of magnitude higher. That is, $M_{\text{string}} \approx g_s \times 5.5 \times 10^{17}$ GeV, where the string coupling $g_s \approx 0.7$. [3] In recent years, three classes of solutions have been proposed to resolve the potential inconsistency between M_{unif} and M_{string} : [4]

- The unification of the MSSM couplings at 2.5×10^{16} GeV should be regarded as a coincidence. M_{unif} could actually be higher as a result of
 1. SUSY–breaking thresholds, [8]
 2. non–MSSM states between 1TeV and M_{unif} , [5]
 3. non–standard hypercharge normalization (a stringy effect), [6, 18] or
 4. non-perturbative effects. [7]
- M_{string} could be lowered by string threshold effects, [3] or
- M_{unif} and M_{string} remain distinct: there is an effective GUT theory between the two scales. MSSM couplings unify around 10^{16} GeV and run with a common value to the string scale.

I have been investigating this third possibility. The rationale for this research has been further strengthened recently by findings suggesting that stringy GUTs and/or non-MSSM states between 1TeV and M_{unif} are the only truly feasible solutions on the list (except perhaps for unknown non-perturbative effects). Shifts upward in M_{unif} from SUSY-breaking and/or non-standard hypercharges appear too small to resolve the conflict and string threshold effects in quasi-realistic models consistently increase M_{string} rather than lower it.[4]

The birth of string GUTs occurred in 1990, initiated in a paper by D. Lewellen.[9] wherein Lewellen constructed a four-generation SO(10) SUSY-GUT built from the free fermionic[10, 11] string. This quickly inspired analysis of constraints on and properties of generic string GUTs.[12, 13, 14] Following this, string GUT research essentially laid dormant until searches for more phenomenologically viable GUTs commenced in 1993 and 1994. Initial results during this second stage of string GUTs seemed to suggest that three generation string-derived GUTs were fairly simple to build and were numerous in number.[15, 16] However, eventually subtle inconsistencies became evident in all these models. The methods used to supposedly yield exactly three chiral generations were inconsistent with worldsheet supersymmetry (SUSY) and, relatedly, unexpected tachyonic fermions were found in the models. The desire to produce three generation SUSY-GUTs consistent with worldsheet SUSY spurred the current stage of string GUT research.[17, 18, 19, 20, 21, 22]

String GUTs and Kač-Moody Algebras

Besides being the possible answer to the $M_{\text{unif}}/M_{\text{string}}$ inconsistency, string GUTs possess several distinct traits not found in non-string-derived GUTs. First, string-derived models can explain the origin of the extra (local) U(1), R, and discrete symmetries often invoked *ad hoc.* in non-string GUTs to significantly restrict superpotential terms.[23]. The extra symmetries in string models tend to suppress proton decay and provide for a generic natural mass hierarchy, with usually no more than one generation obtaining mass from cubic terms in the superpotential. All string GUTs have upper limits to the dimensions of massless gauge group representations that can appear in a given model. Further, the number of copies of each allowed representation is also constrained; there are relationships between the numbers of varying reps that can appear. These features suggest the opportunity for much interplay between string and GUT model builders.

At the heart of string GUTs are Kač-Moody (KM) algebras, the infinite dimensional extensions of Lie algebras.[24] (See Table 1.) A KM algebra may be generated from a Lie algebra by the addition of two new elements to the Lie algebra's Cartan subalgebra (CSA), $\{H^i\}$. These new components are referred to as the "level" K and the "scaling operator" L_0 . K forms the center of the algebra, *i.e.* it commutes with all other members. Therefore, K is fixed for a given algebra in a given string model and is nor-

malized to carry a positive, integer value when the related Lie algebra is non-abelian. L_0 appears automatically in a string model as the zero-mode of the energy–momentum operator. These new elements transform the finite dimensional Lie algebra of CSA and non-zero roots $\{H^i, E^\alpha\}$ into an infinite dimensional algebra, $\{K, L_0, H_m^i, E_m^\alpha\}$, by adding a new indice $m \in \mathbb{Z}$ to the old elements. A KM algebra is essentially an infinite tower of Lie algebras, each distinguished by its m -value.

These KM algebras conspire with conformal and modular invariance (*i.e.* the string self-consistency requirements) to produce tight constraints on string GUTs. There are three generic string-based constraints on gauge groups and gauge group reps. The first specifies the highest allowed level K_i for the i^{th} KM algebra in a consistent string theory. The total internal central charge, c , from matter in the non-supersymmetric sector of a heterotic string must be 22. The contribution, c_i , to this from a given KM algebra is a function of the level K_i of the algebra,

$$c = \sum_i c_i = \sum_i \frac{K_i \dim \mathcal{L}_i}{K_i + \tilde{h}_i} \leq 22 . \quad (1)$$

$\dim \mathcal{L}_i$ and \tilde{h}_i are, respectively, the dimension and dual Coxeter of the associated Lie algebra, \mathcal{L}_i . Eq. (1) places upper bounds of 55, 7, and 4, respectively, on permitted levels of SU(5), SO(10), and E₆ KM algebras.[12, 13]

Once an acceptable level K for a given KM algebra has been chosen, the next constraint specifies what Lie algebra reps could potentially appear. Unitarity requires that if a rep, R , is to be a primary field, the dot product between its highest weight, λ^R , and the highest root of the KM algebra, Ψ , must be less than or equal to K .

$$K \geq \Psi \cdot \lambda^R . \quad (2)$$

For example only the 1, 10, 16, and $\overline{16}$ reps can appear for SO(10) at level 1. (See table 2.) For this reason adjoint Higgs require $K \geq 2$ for SO(10) or any other KM algebra.

Masslessness of a heterotic string state requires that the total conformal dimension, h , of the non-supersymmetric sector of the state equal one. Hence the contribution h_R coming from rep R of the KM algebra can be no greater than one. For a fixed level K , h_R is a function of the quadratic Casimir, C_R , of the rep,

$$h_R = \frac{C_r / \Psi^2}{K + \tilde{h}} . \quad (3)$$

Requiring $h_R \leq 1$ presents a stronger constraint than does unitarity. For instance, although all SO(10) rep primary fields from the singlet up through the 210 are allowed at level 2, only the singlet up through the 54 can be massless. In particular, the 126 cannot be massless unless $K \geq 5$.

Free fermionic string models impose one additional constraint.[18] Increasing the level K decreases the length-squared, Q_{root}^2 , of a non-zero root of the KM algebra by

a factor of K . In free fermionic strings Q_{root}^2 at level 1 is normalized to 2 for the long roots. Thus,

$$KQ_{\text{root}}^2 = 2 . \quad (4)$$

A state containing such a root makes a contribution of $\frac{Q^2}{2} = \frac{1}{K}$ to h . Uncharged free fermionic contributions to h are quantized in units of $\frac{1}{16}$ and $\frac{1}{2}$. Thus, masslessness of gauge bosons constrain K to be a solution of,

$$1 = \frac{1}{K} + \frac{m}{16} + \frac{n}{2}; \quad m, n \in \{0, \mathbb{Z}^+\} , \quad (5)$$

which limits K to values in the set $\{1, 2, 4, 8, 16\}$.

In combination the constraints (1) and (5) permit only levels 1, 2, and 4 for $\text{SO}(10)$ and E_6 , and, in addition to these, also levels 8 and 16 for $\text{SU}(5)$. One result is that massless 126's can never appear in free fermionic $\text{SO}(10)$ SUSY-GUTs; 16's must serve in their stead.

SUSY-GUTs From Free Fermionic Models

In light-cone gauge, a free fermionic heterotic string model[10, 11] contains 64 real worldsheet fermions ψ^m , where $1 \leq m \leq 20$ for left-moving (LM) fermions and $21 \leq m \leq 64$ for right-moving (RM). ψ^1 and ψ^2 are the LM worldsheet superpartners of the two LM scalars embedding the transverse coordinates of four-dimensional spacetime; the remaining ψ^m are internal degrees of freedom.

The transformation property of a real fermion ψ^m around one of the two non-contractible loops of a torus is expressed by $\psi^m \rightarrow -\exp\{\pi i \alpha_m\} \psi^m$, and similarly for the other loop if α_m is replaced by β_m . The α_m and β_m are the m^{th} components of 64-dimensional boundary vectors (BVs) $\vec{\alpha}$ and $\vec{\beta}$, respectively, and have values in the range $(-1, 1]$.

If ψ^m cannot be paired with another real fermion or if it is combined with another to form a Majorana fermion (one LM and one RM fermion), its phases are periodic or antiperiodic, *i.e.* $\alpha_m, \beta_m = 0$ or 1. If a real LM (RM) ψ^m is paired with another real LM (RM) ψ^n to form a Weyl fermion $\psi^{m,n} \equiv \psi^n + i\psi^m$, the phases may be complex (*i.e.* the BV components $\alpha_{m,n} \equiv \alpha_m = \alpha_n$ and $\beta_{m,n} \equiv \beta_m = \beta_n$ may be rational).

A specific model is defined by (1) a set of BVs $\{\vec{\alpha}\}$, describing various combinations of fermion transformations around the two non-contractible loops on the worldsheet torus, and (2) a set of coefficients, $\{C(\frac{\vec{\alpha}}{\vec{\beta}})\}$, weighing the contributions, $Z(\frac{\vec{\alpha}}{\vec{\beta}})$, to the partition function, Z_{ferm} , from the fermions described by each BV pair $(\vec{\alpha}, \vec{\beta})$.

$$Z_{\text{ferm}} = \sum_{\substack{\alpha \in \{\alpha\} \\ \beta \in \{\beta\}}} C\left(\frac{\vec{\alpha}}{\vec{\beta}}\right) Z\left(\frac{\vec{\alpha}}{\vec{\beta}}\right) . \quad (6)$$

The weights $C(\vec{\alpha}/\vec{\beta})$ can be either complex or real (± 1) phases when either $\vec{\alpha}$ or $\vec{\beta}$ have rational, non-integer components, but only real phases when $\vec{\alpha}$ and $\vec{\beta}$ are both integer vectors.

Modular invariance requires that $\{\vec{\alpha}\}$ and $\{\vec{\beta}\}$ be identical sets and that if two vectors, $\vec{\alpha}^i$ and $\vec{\alpha}^j$, are in $\{\vec{\alpha}\}$ then so too is their sum, $\vec{\alpha}^i + \vec{\alpha}^j$. Thus, $\{\vec{\alpha}\}$ and $\{\vec{\beta}\}$ can be defined by choice of some D' -dimensional set of basis vectors $\{\vec{\mathbf{V}}_i\}$,

$$\vec{\alpha} = \sum_{i=1}^{D'} a_i \mathbf{V}_i \pmod{2}, \quad \vec{\beta} = \sum_{i=1}^{D'} b_i \mathbf{V}_i \pmod{2}. \quad (7)$$

Modular invariance also dictates the allowed form of the phase weights:

$$C\left(\frac{\vec{\alpha}}{\vec{\beta}}\right) = (-1)^{s_{\vec{\alpha}} + s_{\vec{\beta}}} \exp\left\{\pi i \sum_{i,j} b_i \left(k_{i,j} - \frac{1}{2} \mathbf{V}_i \cdot \mathbf{V}_j\right) a_j\right\}, \quad (8)$$

where $s_{\vec{\alpha}}$ ($s_{\vec{\beta}}$) is the spacetime component of $\vec{\alpha}$ ($\vec{\beta}$), while $k_{i,j}$ is rational and in the range $(-1, 1]$. There are three mutual constraints on \mathbf{V}_i and $k_{i,j}$:

$$\begin{aligned} k_{i,j} + k_{j,i} &= \frac{1}{2} \mathbf{V}_i \cdot \mathbf{V}_j \pmod{2}, \\ N_j k_{i,j} &= 0 \pmod{2}, \\ k_{i,i} + k_{i,0} &= -s_i + \frac{1}{4} \mathbf{V}_i \cdot \mathbf{V}_i \pmod{2}. \end{aligned} \quad (9)$$

N_j is the smallest positive integer such that $N_j \mathbf{V}_j = 0 \pmod{2}$.

A complex Weyl fermion $\psi^{n,m}$ in a sector $\vec{\alpha}$ carries a $U(1)$ charge $Q_{\alpha}(\psi^{n,m})$ proportional to $\alpha_{m,n}$:

$$Q(\psi^{n,m}) = \alpha_{n,m}/2 + N(\psi^{n,m}). \quad (10)$$

N is the fermion number operator and has eigenvalues $0, \pm 1$. Each $\vec{\alpha}$ yields a set of states that are excitations of the vacuum by various modes of the real $\{\psi^{m'}\}$ or complex $\{\psi^{m,n}\}$. These states, therefore, carry differing charge vectors \mathbf{Q}_{α} . Together, the charges of all states in all sectors form a lattice upon which the roots and weights of an algebra can be embedded. The BVs $\vec{\beta}$ contribute a set of GSO operators that project out certain states in each sector: for a state in a sector $\vec{\alpha} = a_i \mathbf{V}_i$ to survive, its charge vector \mathbf{Q}_{α} must separately satisfy the relation,

$$\mathbf{V}_j \cdot \mathbf{Q}_{\alpha} = \left(\sum_i k_{j,i} a_i\right) + s_j \pmod{2}, \quad (11)$$

for each basis vector \mathbf{V}_j .

Consider now level- K $SO(10)$ (henceforth denoted $SO(10)_K$) models. As the prior section showed, the only allowed levels are 1, 2, and 4 corresponding to $Q_{\text{root}}^2 = 1, 1/2$, and $1/4$, respectively. Level-2 and level-4 algebras require charge lattices of dimension greater than the rank of $SO(10)$, *i.e.* more than five associated $U(1)$ charges

are required for each embedding. For example, the minimal level-2 embedding requires a six-dimensional charge lattice, with charge vectors for the five $\text{SO}(10)$ simple roots given by $(0, 0, 0, 1, 0, 0)$, $(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, 0, 0)$, $(0, 0, 1, 0, 0, 0)$, $(0, \frac{1}{2}, -\frac{1}{2}, 0, -\frac{1}{2}, \frac{1}{2})$, and $(0, \frac{1}{2}, -\frac{1}{2}, 0, \frac{1}{2}, -\frac{1}{2})$. The extra degree of freedom on the lattice corresponds to an additional $\text{U}(1)$ algebra.

Although the total central charge for $\text{SO}(10)_2 \times \text{U}(1)$ is 10, the charge lattice for $\text{SO}(10)_2 \times \text{U}(1)$ only yields a central charge of 6 (since each complex fermion contributes 1 to the central charge). Additional central charge must come from unpairable real fermions (URFs), *i.e.* real fermions that cannot form Weyl or Majorana fermions.[10] URFs assume the role of increasing the central charge without increasing the number of local $\text{U}(1)$ charges. It is in this manner that free fermions can match the effect of increasing the level of a KM algebra.

Lewellen demonstrated that the smallest possible URF set is formed from 16 real fermions (containing a central charge of 8). This set can contribute half of its central charge to realize the required $\text{SO}(10)_2 \times \text{U}(1)$ central charge of 10. (Existence of a remaining URF central charge of $4 = 8 - 4$ denotes the presence of a discrete symmetry among the URFs.) Lewellen's $\text{SO}(10)_2$ embedding presents an example of how free fermionic representations of higher level KM algebras involve both charged and uncharged sectors. All of the “second stage” attempts at three generation $\text{SO}(10)_2$ models[15, 16] involved both the minimal (six-dimensional) $\text{SO}(10)_2 \times \text{U}(1)$ charge embedding and the minimal $c = 8$ URF set. One direction of my current research is to proceed beyond these minimal embeddings and comprehensively classify and investigate the further possible $\text{SO}(10)_2$ charged and uncharged embeddings. Each physically inequivalent choice of charged and uncharged embeddings should define a new class of $\text{SO}(10)_2$ models. When my investigation of $\text{SO}(10)_2$ models is complete, I will proceed on with similar treatment of $\text{SO}(10)_4$. One important requirement for all such models is that they have $N = 1$ spacetime SUSY, which is the topic of the next section.

Classes of N=1 Spacetime SUSY Models

In D -dimensional heterotic free fermionic models, the $3(10 - D)$ real internal LM fermions (henceforth denoted by χ^I rather than ψ^m) non-linearly realize a worldsheet SUSY through a supercurrent of the form[28, 10]

$$T_F = \psi^\mu \partial X_\mu + f_{IJK} \chi^I \chi^J \chi^K. \quad (12)$$

The f_{IJK} are the structure constants of a semi-simple Lie algebra \mathcal{L} of dimension $3(10 - D)$. Four-dimensional models can involve any one of the three 18-dimensional Lie algebras: $\text{SU}(2)^6$, $\text{SU}(2) \otimes \text{SU}(4)$, and $\text{SU}(3) \otimes \text{SO}(5)$. When T_F is transported around the non-contractible loops on the worldsheet, it must transform identically as ψ^μ does: periodically for spacetime fermions and antiperiodically for spacetime bosons. This requirement severely constrains the BVs in consistent models. Since $f_{IJK} \chi^I \chi^J \chi^K$ must

transform as ψ^μ does, each BV necessarily represents an automorphism (up to a minus sign) of the chosen algebra.

The simplest such four-dimensional modular invariant model is non-supersymmetric. Its single basis vector is the all-periodic \mathbf{V}_0 ; therefore it contains only the sectors \mathbf{V}_0 and $\vec{0} \equiv \mathbf{V}_0 + \mathbf{V}_0$ (the all antiperiodic sector). The graviton, dilaton, antisymmetric tensor, and spin-1 gauge particles all originate from the $\vec{0}$ sector. Each of the three possible worldsheet SUSY choices for Lie algebra allows various possibilities for an additional basis vector \mathbf{S}_i that both satisfies the automorphism constraint and contributes massless gravitinos. Every $\{\mathbf{V}, \mathbf{S}_i\}$ set generates an $N = 4$ supergravity model. Additional basis vectors (with related GSO projections) must be added to reduce the number of spacetime supersymmetries below four. Ref. [30] showed that neither $SU(2) \otimes SU(4)$ nor $SU(3) \otimes SO(5)$ algebras can be used to obtain $N = 1$ spacetime SUSY. This work also presented two examples of different basis vector combinations (one being the NAHE set of LMs)[26] that can yield $N = 1$ for $SU(2)^6$, while it revealed one situation where presence of a specific basis vector forbids $N = 1$.

I have finished the work initiated in [30]. That is, I have *completely* classified the sets of LM BVs that can produce exactly $N = 1$ spacetime SUSY (and $N = 4, 2$, and 0 spacetime SUSY solutions in the process). $SU(2)^6$ is necessarily the supercurrent's Lie algebra, which gives (12) the form of,

$$T_F = \psi^\mu \partial X_\mu + i \sum_{J=1}^6 \chi^{3J} \chi^{3J+1} \chi^{3J+2}. \quad (13)$$

Each fermion triplet $(\chi^{3J}, \chi^{3J+1}, \chi^{3J+2})$ represents the three generators of the J^{th} $SU(2)$. $N = 1$ spacetime is only possible if the generators for each $SU(2)$ are written in the non-Cartan-Weyl basis of $(J_3, J_1, \text{ and } J_2)$.

An automorphism of $SU(2)^6$ is the product of inner automorphisms for the separate $SU(2)$ algebras and an outer automorphism of the whole $SU(2)^6$ product algebra.[11, 30] The only inner automorphism for an individual $SU(2)$ that could yield a massless gravitino corresponds to one fermion in a triplet being periodic and the other two being antiperiodic. An outer automorphism can be expressed as an element of the permutation group P_6 that mixes the $SU(2)$ algebras.[30] The elements of P_6 can be resolved into factors of disjoint commuting cycles. These fit into eleven classes defined by the different possible lengths, n_k , of the cycles in the permutation such that $\sum_k n_k = 6$. The set of these eleven classes (with a set of lengths written as $n_1 \cdot n_2 \cdots n_i$) is

$$\mathbf{n} \in \{ 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1, 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1, 2 \cdot 2 \cdot 1 \cdot 1, 2 \cdot 2 \cdot 2, \\ 3 \cdot 1 \cdot 1 \cdot 1, 3 \cdot 2 \cdot 1, 3 \cdot 3, 4 \cdot 1 \cdot 1, 4 \cdot 2, 5 \cdot 1, 6 \}. \quad (14)$$

The first element in this set, $1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$, is the P_6 identity element, while $2 \cdot 1 \cdot 1 \cdot 1 \cdot 1$ is the class with cyclic permutation between two $SU(2)$ algebras (which two is indicated by each class member's J subscripts). For example,

$$2_{1,2} \cdot 1 \cdot 1 \cdot 1 \cdot 1 : (\chi^3, \chi^4, \chi^5), \leftrightarrow (\chi^6, \chi^7, \chi^8). \quad (15)$$

Similarly, an element of the $2 \cdot 2 \cdot 1 \cdot 1$ class permutes two separate pairs of algebras, *e.g.*

$$2_{1,2} \cdot 2_{3,4} \cdot 1 \cdot 1 : \quad (\chi^3, \chi^4, \chi^5) \leftrightarrow (\chi^6, \chi^7, \chi^8), \quad (16)$$

$$(\chi^9, \chi^{10}, \chi^{11}) \leftrightarrow (\chi^{12}, \chi^{13}, \chi^{14}). \quad (17)$$

Of the eleven permutation classes, only those six involving an even number of disjoint permutations correspond to BVs that can yield massless gravitinos.[30] The other five would produce gravitino BVs that cannot satisfy all requirements of (9). The six distinct gravitino BVs are listed in Table 3. (Note that, as with any BV, a \mathbb{Z}_n twisted gravitino generator contains components of the form $\frac{2a}{n}$ where a and n are relative primes in at least one component.) I have studied each gravitino generator and applied all consistent combinations of unique GSO projections to it.[17] I have determined how many of the initial $N = 4$ spacetime SUSYs survive various combinations of GSO projections. My findings can be summarized as follows:

1. Only left-moving \mathbb{Z}_2 , \mathbb{Z}_4 , and \mathbb{Z}_8 twists that correspond to automorphisms of $SU(2)^6$ are consistent with $N = 1$ in free fermionic models. All other LM \mathbb{Z}_n twists obviate $N = 1$. Thus, neither gravitino generators \mathbf{S}_5 and \mathbf{S}_7 (both containing \mathbb{Z}_6 twists), nor \mathbf{S}_{10} (containing \mathbb{Z}_{10} twists) can produce $N = 1$ spacetime SUSY. \mathbf{S}_5 and \mathbf{S}_7 only result in $N = 4, 2$, or 0 , whereas \mathbf{S}_{10} yields $N = 4$ or 0 .
2. $N = 1$ spacetime SUSY is possible for \mathbf{S}_1 , \mathbf{S}_3 , and \mathbf{S}_9 . Six general categories of GSO projection sets lead to $N = 1$ for \mathbf{S}_1 , while three do for \mathbf{S}_3 , and one does for \mathbf{S}_9 . The GSO projections in all of these sets originate from LM BVs with the above-mentioned \mathbb{Z}_2 , \mathbb{Z}_4 , and \mathbb{Z}_8 twists.

I have fully classified the ways by which the number of spacetime supersymmetries in heterotic free fermionic strings may be reduced from $N = 4$ to the phenomenologically preferred $N = 1$. This means that the set of LM BVs in any free fermionic model with claimed $N = 1$ spacetime SUSY must be reproducible from one of the three specific gravitino sectors in the set $\{\mathbf{S}_1, \mathbf{S}_3, \mathbf{S}_9\}$, combined with one of my LM BV sets whose GSO projections reduce the initial $N = 4$ to $N = 1$. The only variations from my BVs that true $N = 1$ models could have (besides trivial reordering of BV components) are some component sign changes that I have shown do not lead to physically distinct models.

Prior to my present $SO(10)_2$ research, only the gravitino generator \mathbf{S}_1 had been used in $N = 1$ models. Reduction to $N = 1$ spacetime SUSY had always been accomplished through GSO projections from the NAHE set of LM BVs.[26] Thus, my new $N = 1$ solutions should be especially useful for model building when the NAHE set may be inconsistent with other properties specifically desired in a model. This, indeed, appears to be the situation with regard to current searches for consistent three generation free fermionic $SO(10)$ level-2 models, at least when Lewellen's original minimal charged and uncharged embeddings are chosen.

Concluding Comments

The result of the 1993–1994 second stage of the search for string-derived three generation $\text{SO}(10)$ SUSY-GUTs was essentially a free fermionic no-go theorem for a particular choice of charged and uncharged $\text{SO}(10)_2$ embeddings that was combined with the standard gravitino generator, \mathbf{S}_1 . Free fermionic string GUT research has in 1995 advanced to a more mature stage, with classification of non-minimal charged and uncharged embeddings now underway. Further, complete classification of all directions to obtaining $N = 1$ spacetime SUSY has been completed. Relatedly, new classes of $\text{SO}(10)_2$ free fermionic models are now under investigation. In parallel fashion, $\text{SO}(10)_4$ models will also be examined. If three generation free fermionic $\text{SO}(10)$ SUSY-GUT models do exist, they will eventually be found through the systematic search now in operation.

Note added:

Since completion of this paper in March, 1996, significant advancement has been made in the more general field of higher-level group theoretic embeddings.[29] (The free fermionic approach is one method by which such embeddings can be realized.) The recent work discussed in [29] has led to full classification of group theoretic embeddings of $\text{SU}(5)$, $\text{SU}(6)$, $\text{SO}(10)$, and E_6 in strings. One result of this is a generalized proof that heterotic strings cannot yield massless **126** representations of $\text{SO}(10)$.

Table 1. Kač–Moody Algebras –vs– Lie Algebras

LIE ALGEBRA with rank l :

- FINITE dimensional algebra

$$\begin{aligned}
 [H^i, H^j] &= 0; \quad i, j \in \{1, 2, \dots, l\} \\
 [H^i, E^\alpha] &= \alpha(H^i) E^\alpha \\
 [E^\alpha, E^\beta] &= \begin{cases} \epsilon(\alpha, \beta) E^{\alpha+\beta}, & \text{if } \alpha + \beta \text{ is a root;} \\ \frac{2}{\alpha^2} \alpha \cdot H, & \text{if } \alpha + \beta = 0; \\ 0, & \text{otherwise.} \end{cases} \quad (18)
 \end{aligned}$$

AFFINE KAC-MOODY ALGEBRA with rank $l + 2$:

- New elements in CSA are “LEVEL” K (center of group) and “scaling/energy operator” L_0
- INFINITE dimensional algebra: $m, n \in \mathbb{Z}$

$$\begin{aligned}
 [H_m^i, H_n^j] &= Km \delta^{ij} \delta_{m, -n}; \quad i, j \in \{0, 2, \dots, l+1\} \\
 [H_m^i, E_n^\alpha] &= \alpha(H_0^i) E_{m+n}^\alpha \\
 [E_m^\alpha, E_n^\beta] &= \begin{cases} \epsilon(\alpha, \beta) E_{m+n}^{\alpha+\beta}, & \text{if } \alpha + \beta \text{ is a root;} \\ \frac{2}{\alpha^2} [\alpha \cdot H_{m+n} + Km \delta_{m, -n}], & \text{if } \alpha + \beta = 0; \\ 0, & \text{otherwise.} \end{cases} \\
 [K, H_m^p] &= [K, E_m^\alpha] = 0 \\
 [L_0, H_m^p] &= -m H_m^p \\
 [L_0, E_m^\alpha] &= -m E_m^\alpha
 \end{aligned}$$

Table 2. Potentially Massless Unitary Gauge Group Reps

	$k = 1$	$k = 2$	$k = 3$	$k = 4$
SU(5)	$c = 4$	$c = 48/7$	$c = 9$	$c = 32/3$
	rep h	rep h	rep h	rep h
	5 2/5	5 12/35	5 3/10	5 4/15
	10 3/5	10 18/35	10 9/20	10 2/5
		15 4/5	15 7/10	15 28/45
		24 5/7	24 5/8	24 5/9
		40 33/35	40 33/40	40 11/15
		45 32/35	45 4/5	45 32/45
			75 1	50 14/15
				70 14/15
				75 8/9
SO(10)	$c = 5$	$c = 9$	$c = 135/11$	$c = 15$
	rep h	rep h	rep h	rep h
	10 1/2	10 9/20	10 9/22	10 3/8
	16 5/8	16 9/16	16 45/88	16 15/32
		45 4/5	45 8/11	45 2/3
		54 1	54 10/11	54 5/6
			120 21/22	120 7/8
			144 85/88	144 85/96
				210 1

Table 3. Distinct Free Fermionic Gravitino Boundary Vectors

<u>BV Class</u>	<u>Gravitino Boundary Vectors</u>	<u>Allowed SUSY</u>
$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$	$\mathbf{S}_1 = \{1, 1 \quad (1; 0, 0)^6\}$	4, 2, 1, 0
$2 \cdot 2 \cdot 1 \cdot 1$	$\mathbf{S}_3 = \{1, 1 \quad (0, 1; -\frac{\hat{1}}{2}, \frac{\hat{1}}{2})^2 \quad (1; 0, 0)^2\}$	4, 2, 1, 0
$3 \cdot 1 \cdot 1 \cdot 1$	$\mathbf{S}_5 = \{1, 1 \quad (\frac{\hat{1}}{3}, 1; -\frac{\hat{2}}{3}, 0, 0, \frac{\hat{2}}{3}) \quad (1; 0, 0)^3\}$	4, 2, 0
$3 \cdot 3$	$\mathbf{S}_7 = \{1, 1 \quad (\frac{\hat{1}}{3}, 1; -\frac{\hat{2}}{3}, 0, 0, \frac{\hat{2}}{3})^2\}$	4, 2, 0
$4 \cdot 2$	$\mathbf{S}_9 = \{1, 1 \quad (0, \frac{\hat{1}}{2}, 1; -\frac{\hat{3}}{4}, -\frac{\hat{1}}{4}, \frac{\hat{1}}{4}, \frac{\hat{3}}{4}) \quad (0, 1; -\frac{\hat{1}}{2}, \frac{\hat{1}}{2})\}$	4, 2, 1, 0
$5 \cdot 1$	$\mathbf{S}_{10} = \{1, 1 \quad (\frac{\hat{1}}{5}, \frac{\hat{3}}{5}, 1; -\frac{\hat{4}}{5}, -\frac{\hat{2}}{5}, 0, 0, \frac{\hat{2}}{5}, \frac{\hat{4}}{5}) \quad (1; 0, 0)\}$	4, 0

*

References

- [1] M. Green, J. Schwarz, and E. Witten, Superstring Theory, **Vols. I & II**, (Cambridge University Press, New York, 1987);
M. Kaku, Introduction to Superstrings, (Springer-Verlag, New York, 1988).
- [2] P. Langacker, *Particle Physics Summary: Where Are We And Where Are We Going*, UPR-0512-T (1992); hep-ph/9207201. In the Proceedings of Moriond 1992.
- [3] V. Kaplunovsky, *Nucl. Phys.* **B307** (1988) 145;
V. Kaplunovsky and J. Louis, “*On Gauge Couplings in String Theory*,” hep-th/9502077; UTTG-24-94; LMU-TPW-94-24.
- [4] For a review of the feasibility of the various proposed solutions and a more complete list of related references see K. Dienes, “*String Theory and the Path to Unification: A Review of Recent Developments*” IASSNS-HEP-95/97; hep-th/9602045. See also:
K. Dienes and A. Faraggi, *Phys. Rev. Lett.* **75** (1995) 2646; *Nucl. Phys.* **B457** (1995) 409;
K. Dienes et. al, *Nucl. Phys.* **B467** (1996) 44.
- [5] A. Faraggi, *Phys. Lett.* **B302** (1993) 202.
- [6] J. Casas and C. Muñoz, *Phys. Lett.* **B214** (1988) 543;
L. Ibanez, *Phys. Lett.* **B318** (1993) 73;
H. Kawabe, et. al, *Nucl. Phys.* **B434** (1995) 210;
K. Dienes, et. al, *String Unification, Higher Level Gauge Symmetries and Exotic Hypercharge Normalizations*, hep-th/9510223;
B. Allanach and S. King, “*String Gauge Unification and Infrared Fixed Points in MSSM + X Models*,” hep-th/9601391.
- [7] E. Witten, “*Strong Coupling Expansion of Calabi-Yau Compactifications*,” IASSNS-HEP-96/08; hep-th/9602070.
- [8] K. Dienes and A. Faraggi, *Phys. Rev. Lett.* **75** (1995) 2646; *Nucl. Phys.* **B457** (1995) 409.
- [9] D. Lewellen, *Nucl. Phys.* **B337** (1990) 61.
- [10] H. Kawai, et. al, *Nucl. Phys.* **B288** (1987) 1; *Nucl. Phys.* **B299** (1988) 431.
- [11] I. Antoniadis, et. al, *Nucl. Phys.* **B289** (1987) 87; *Nucl. Phys.* **B298** (1988) 586.
- [12] A. Font, et. al, *Nucl. Phys.* **B345** (1990) 389.

- [13] J. Ellis, *et. al*, *Phys. Lett.* **B245** (1990) 375.
- [14] J. Schwartz, *Phys. Rev.* **D42** (1990) 1777.
- [15] G. Cleaver, “*Guts with Adjoint Higgs from Superstrings*”, in the proceedings of PASCOS '94, May 1994, Syracuse, New York, p. 223; “*SO(10) SUSY-GUTs Based on Superstrings*,” OHSTPY-HEP-T-94-008. In the Proceedings of DPF '94, August, 1994, Albuquerque, New Mexico, p. 1442.
- [16] Chaudhuri, *et. al*, “*Fermion Masses from Superstrings with Adjoint Scalars*,” Fermilab-PUB-94/137-T; “*String Models for Locally Supersymmetric Grand Unification*,” hep-th/9409151. In the Proceedings of DPF '94, August, 1994, Albuquerque, New Mexico, p. 1393.
- [17] G. Cleaver, *Nucl. Phys.* **456** (1995) 219; *What's New in Stringy SO(10) SUSY-GUTs*,” hep-th/9506006. In the Proceedings of Strings '95, March 1995, Los Angeles, California.
- [18] S. Chaudhuri, *et. al*, *Nucl. Phys.* **456** (1995) 89; “*Three Generations in Fermionic Construction*,” Fermilab-PUB-95/349-T.
- [19] Aldazabal, *et. al*, “*Standard Grand Unification from Superstrings*,” hep-th/9507162, in the Proceedings of SUSY '95, Paris, May 1995; *Nucl. Phys.* **452** (1995) 3; “*Building GUTs from Strings*,” FTUAM-95-27.
- [20] Z. Kakushadze and S.H. Tye, CLNS-95-1380; hep-th/9512156.
- [21] J. Erler, SCIPP-96-10; hep-th/9512156.
- [22] For general discussions of three generations in free fermionic models see also:
A. Faraggi, *Phys. Lett.* **B326** (1994) 62;
A. Faraggi, and D.V. Nanopoulos, *Phys. Rev.* **D48** (1993) 3288-3296.
- [23] L. Hall and S. Raby, *Phys. Rev.* **D51** (1995) 6524;
Anderson *et. al*, *Phys. Rev.* **D49** (1994) 3660.
- [24] V. Kač, Infinite Dimensional Lie Algebras, (Birkhäuser, Boston, 1983);
V. Kač editor, Infinite Dimensional Lie Algebras and Groups, (World Scientific, Singapore, 1989).
- [25] H. Dreiner, *et. al*, *Nucl. Phys.* **B320** (1989) 401.
- [26] A. Faraggi, *Nucl. Phys.* **B387** (1992) 239;
I. Antoniadis, *et. al* *Phys. Lett.* **194** (1987) 231.
- [27] P. Candelas, *et. al*, *Nucl. Phys.* **B258** (1985) 46.

- [28] I. Antoniadis, *et. al*, *Phys. Lett.* **B149** (1987) 231.
- [29] K. Dienes and J. March-Russell, *Realizing Higher-Level Gauge Symmetries in String Theory: New embeddings for String GUTs*, hep-th/9604112.
 For an introduction to the general field of higher-level group theoretic embeddings in bosonic or heterotic strings see also:
 F. Bais *et. al*, *Nucl. Phys.* **B279** (1987) 529;
 F. Bais and P. Bouwknecht, *Nucl. Phys.* **B279** (1987) 561.
- [30] H. Dreiner, *et. al*, *Phys. Lett.* **B216** (1989) 283.